Impact of Mobility on Energy Provisioning in Wireless Rechargeable Sensor Networks

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Abstract—One fundamental question in Wireless Rechargeable Sensor Networks (WRSNs) design is how to deploy energy sources in a network to ensure that nodes within it can harvest sufficient energy for continuous operation, which is referred as energy provisioning problem in previous works. Though the potential mobility of nodes is exploited to reduce the number of sources necessary in path energy provisioning problem, we argue that sustainable operation of nodes may not be guaranteed in this case by taking into consideration the constraints of node speed and battery capacity.

In this paper, we propose a new metric — Quality of Energy Provisioning (QoEP) — to characterize the expected portion of time that a mobile node can sustain normal operation in WRSNs. To avoid the limitation of analysis in a specific mobility model, we study spatial distribution instead. We investigate the upper and lower bounds of QoEP in one-dimensional case with one single source and multiple sources respectively. For single source case, we not only show that exact QoEP can be found in some special cases, but also prove the tight lower bound and upper bound of QoEP. In particular, we present a novel analytical approach — flow pattern analysis to design an optimal mobility model which achieves the largest QoEP. Then we propose an algorithm to calculate the upper bound. Extending the results to multiple sources, we obtain tight lower bound and relaxed upper bound in normal cases, together with tight upper bound for one special case. Moreover, we find the tight lower bounds in both 2D and 3D cases. Finally, we perform extensive simulations to verify our findings. Simulation results show that our bounds perfectly hold, and outperform the former works.

Keywords—wireless rechargeable sensor network; mobility; energy provisioning

I. INTRODUCTION

Wireless Sensor Networks (WSNs) are mainly powered by small batteries, and the limited energy supply constrains the lifetime of WSNs. Recent studies have shown that energy harvesting wireless sensor networks have the potential to provide perpetual network operations by capturing renewable energy from the ambient environment. However, vibrations of the ambient environment make the availability of the energy unpredictable and uncontrollable, and consequently limit the success of energy harvesting-based approaches.

The emergence of wireless power charging technology [1] has shed light on the power supply problem in WSNs. By using the wireless charging technology, we can create a controllable and perpetual energy source to provide wireless power from a distance. Tong et al. [2] investigated the impact of wireless charging technology on sensor network deployment and routing arrangement, when both the sensor and source are stationary. The authors in [3] and [4] used a wireless charging vehicle (WCV) or a mobile charger (MC) to periodically travel inside the sensor network and charge each static sensor node’s battery via wireless, and studied the optimization problem about routing and charging. In [5], a mobile charger was used to serve not only as an energy transporter that charges static sensors, but also as a data collector.

Despite the fact that many schemes have been proposed to make use of wireless charging, little literature focuses on the issues with mobile nodes. The role of mobility in Wireless Rechargeable Sensor Networks (WRSNs) has been largely overlooked, whereas a number of studies have illustrated that mobility enhances the capacity [6], increases the connectivity [7], and brings coverage improvement [8] in WSNs.

Nevertheless, He et al. [9] investigated the energy provisioning problem in a WRSN built from the industrial Wireless Identification and Sensing Platform (WISP) [10]. Specifically, two forms of the problem are studied. That is, point (energy) provisioning and path (energy) provisioning, depending on how to deploy readers in a network to ensure that static and mobile tags can harvest sufficient energy for continuous operation. In detail, the path energy provisioning requires that the average recharge rate is no smaller than the power consumption of tags in the long run. However, this condition can not necessarily guarantee sustainable operation for tags, since recharge energy loss will happen if a tag travels into a power-rich area with fully charged battery, which would eventually result in energy shortage and operation suspension in power-deficient areas. This observation shows that battery capacity limits the performance of energy provisioning.

Besides, node speed also plays an important role in energy provisioning performance. Generally, if a node moves with a relatively high speed, it will be able to effectively avoid
recharge energy loss in power-rich areas and free from energy depletion in power-deficient areas accordingly, as each of its residence time in these areas is comparatively short.

To evaluate the performance of energy provisioning for mobile nodes, we propose a new metric, namely Quality of Energy Provisioning (QoEP), which is defined as the expected portion of time that a node can sustain normal operation. It captures the characteristics of energy provisioning performance even when node works in an intermittent mode. In this paper, our work is mainly focused on the upper and lower bounds of QoEP based on node spatial distribution. Theoretical results provide insights into how to effectively deploy energy sources to meet requirements for energy provisioning performance in applications, while factoring in constraints of node speed and battery capacity. Generally, we make the following main contributions.

- For the single source case in one-dimensional networks, we obtain exact QoEPs in two special cases and the tight lower bound in general cases after the formal definition of QoEP in one-dimensional case. Further, we prove the tightness of the lower bound.
- To analyze the upper bound of QoEP in single source case, we propose a totally new analytical approach, called flow pattern analysis, which is designed by drawing an analogy to flow in physics, and construct an optimal mobility model which achieves the highest QoEP and prove its optimality.
- We obtain an upper bound of QoEP and prove its tightness. We present an effective algorithm to calculate the QoEP of the optimal mobility model subsequently, by leveraging the energy conservation nature in flow pattern.
- We further extend the results to multiple sources, and derive tight lower bound and relaxed upper bound in general cases. More importantly, we also present the tight lower bounds in both 2D and 3D cases.
- We conduct simulations mainly in two cases: the random waypoint mobility model in single source case and the random walk mobility model in multiple sources case. Simulation results show that our upper and lower bounds perfectly hold for these models, and outperform the path energy provisioning solution.

The rest of the paper is organized as follows. In Section II, the problem definition and related conceptions are stated. Section III contains our main results in single source case where the tight lower bound and upper bound for WRSNs are studied. We extend our results in multiple sources in section IV. Then in Section V we give the results of the simulations to support our theoretical findings. Finally, we conclude this work in Section VI.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tr>
<td>$v_{\text{max}}$</td>
<td>The maximum speed of the node.</td>
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<tr>
<td>$p_s$</td>
<td>The constant nodal power consumption for working.</td>
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<tr>
<td>$E_b$</td>
<td>The battery capacity of node.</td>
</tr>
<tr>
<td>$p_r(t)$ ($p_r(x)$)</td>
<td>The cumulative recharge power node receives at time $t$ (location $x$).</td>
</tr>
<tr>
<td>$E_{\text{res}}(t)$ ($E_{\text{res}}(x)$)</td>
<td>The residual energy at time $t$ (location $x$).</td>
</tr>
<tr>
<td>$f_{\text{dis}}(x)$</td>
<td>The spatial distribution at location $x$.</td>
</tr>
<tr>
<td>$\lambda_c(x)$ ($\Lambda_c(\Omega)$)</td>
<td>The expected battery energy consumption rate for location $x \in \Omega_o$ (subregion $\Omega' \subseteq \Omega_o$).</td>
</tr>
<tr>
<td>$\lambda_h(x)$ ($\Lambda_h(\Omega)$)</td>
<td>The expected battery energy harvest rate for location $x \in \Omega_i$ (subregion $\Omega' \subseteq \Omega_i$).</td>
</tr>
<tr>
<td>$p_{\text{ep}}(x)$</td>
<td>The energy providing ability for location $x \in \Omega_o$.</td>
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II. PROBLEM STATEMENT

In this paper, we use a wireless recharge infrastructure which is much similar to [9]. That is, a WRSN consists of a single or multiple static sources and a set of mobile nodes (e.g., they are worn by human users for activity monitoring) in a region of interest. Sources are responsible for recharging the nodes via wireless. Nodes harvest wireless energy and store it in their batteries for normal operations like sensing and logging data. One objective proposed by [9] is to keep nodes’ sustainable operation to avoid any information loss, which is formalized as energy provisioning problem. By contrast, the data reporting process is supposed to be delay-tolerant and occur in a batch manner thus can be ignored.

This wireless recharge infrastructure is generic and can be reused for diverse types of different applications. For example, we can utilize equipment from Powercast [11], involving power transmitters and rechargeable sensor nodes, to build a WRSN, in which power transmitters continuously send out RF radio in frequency 903-927 MHz to rechargeable sensor nodes. Clearly, it also fits our proposed infrastructure.

For clarity of presentation, we list the notations used in this paper in Table I. We further propose some assumptions about node mobility and its energy consumption as follows.

(i) At any time, a node can move at an arbitrary speed which is no more than $v_{\text{max}}$; (ii) The node’s power consumption for working, such as sensing and logging data, is constant and independent of its motion, which can be denoted as $p_s$; (iii) Each node has a battery capacity of $E_c$, and its power leakage of battery can be neglected. Moreover, a node keeps working with a nonzero residual energy till depletion, and then it suspends its work. As long as the node absorbs any amount of energy, it resumes work immediately. The energy cost for switching on or off can be ignored.

In some applications nodes are scheduled to sense and log data for some time, and sleep regularly to save energy in a cycled fashion, then we use the corresponding average energy power $\overline{p_s}$ instead of $p_s$. Moreover, in common practices, thresholds of residual energy are usually set to
guide the node to switch to work state or sleep state. For this situation, we can approximate it by appropriately reducing the node battery capacity.

A critical factor impacting energy provisioning is the wireless recharge model. In this paper, we adopt the practical models assumed by [9] when either a single source or multiple sources are used in WRSN. In particular, the receive power \( p_r \) of RF signal, \( d \) m away from the source can be expressed as \( p_r = \frac{\tau}{(\tau + \beta)^2} \), where both \( \tau \) and \( \beta \) are constants independent of \( d \). More information about this model can be found in [9]. In addition, for multiple sources, the additivity of the transmission power of multiple sources is verified by realistic experiment. That is, the recharge power at a specific location is simply the sum of the individual recharge power of each source. What’s more, the mobility of node on recharge power brings positive effects that make the average experimental recharge power fits the model better. As a result, we can neglect its impact in our theoretical analysis.

Before formally stating the problem in this paper, we introduce the following definitions in advance.

**Definition 2.1:** Instantaneous Quality of Energy Provisioning (IQoEP) of node at time \( t \) is defined as follows:

\[
IQoEP(t) = \begin{cases} 
\frac{p_r(t)}{p_r}, & E_{re}(t) = 0 \text{ and } p_r(t) < p_s \\
1, & \text{otherwise}
\end{cases}
\]

where \( p_r(t) \) is the cumulative recharge power node receives at time \( t \), and \( E_{re}(t) \) is the residual energy at that moment.

The above equation is directly derived from the third assumption. Then we propose our new metric.

**Definition 2.2:** Node’s Quality of Energy Provisioning (QoEP) in the region \( \Omega \) is defined as the expected portion of time in the long run that a node can sustain normal operation. Hence:

\[
QoEP = \lim_{t \to \infty} \frac{1}{t - t_0} \int_{t_0}^{t} IQoEP(t')dt'
\]

We note that this concise form of QoEP not only simplifies our following analysis, but also captures the characteristics of energy provisioning performance, even when node works in an intermittent mode. In addition, QoEP provides insights into how to effectively deploy energy sources to meet requirements for energy provisioning in applications, while factoring in constraints of node speed and battery capacity.

**Definition 2.3:** Node’s Quality of Energy Provisioning at Location \( x \) (LQoEP) is defined as the expected proportion of cumulative time that a mobile node can sustain normal operation to that node spent at \( x \).

\[
LQoEP(x) = \lim_{t \to \infty} \frac{\int_{t_0}^{t} IQoEP(t')I(t', x)dt'}{\int_{t_0}^{t} I(t', x)dt'}
\]

where \( I(t', x) \) is an indicator function with the value 1 when node appears at location \( x \) at time \( t' \), or 0 otherwise.

With \( LQoEP(x) \), Equation (2) can be rewritten as:

\[
QoEP = \lim_{t \to \infty} \frac{1}{t - t_0} \int_{t_0}^{t} IQoEP(t')I(t', x)dt'
\]

\[
= \lim_{t \to \infty} \frac{1}{t - t_0} \int_{\Omega} \int_{t_0}^{t} IQoEP(t')I(t', x)dt'dx
\]

\[
= \lim_{t \to \infty} \int_{\Omega} LQoEP(x) \frac{\int_{t_0}^{t} I(t', x)dt'}{t - t_0} dx
\]

\[
= \int_{\Omega} LQoEP(x) f_{dis}(x)dx
\]

Intuitively, there may exist multiple mobility models obeying a same spatial distribution \( f_{dis}(x) \). Denote \( P \) the set of all those qualified mobility models, we define the lower bound of \( QoEP \) with respect to spatial distribution \( f_{dis}(x) \) by \( QoEP_{min} \), which is less than or equal to any \( QoEP \) of mobility model in \( P \). We define the upper bound of \( QoEP \) by \( QoEP_{max} \) in a similar way. Actually, in this paper we prefer to investigate the impact of nodal mobility on energy provisioning based on spatial distribution rather than a specific mobility model, with the following three issues concerned.

First, there have emerged so many mobility models. These models can be roughly divided into two kinds, namely empirical ones which modeling mobility based on some empirical evidences such as WLAN trace [12] and survey data [13], and theoretical ones based on studies of the human behavior [14] or mathematically tractable models like random waypoint mobility model [15], as [16] summarized. Hence analysis in spatial distribution is undoubtedly more general compared with that in a specific mobility model. Second, a vast of theoretical mobility models are demonstrated to be “non-realistic” in [17] and suffer from subtle problems such as decay and instability [18]. On the contrary, solutions based on spatial distributions are stable, and also practical for they can utilize the primary data directly which eliminate the error caused by mobility modeling. Third, as the recharging process of a node is determined by its location and battery capacity, the calculation of QoEP based on mobility models becomes quite difficult, let alone for those mobility models with obstacles [19], reflection or wrapping, city section, and space graph [20]. Despite of this, spatial distribution can still be obtained through Palm calculus [21], knowledge of the specific mobility patterns, empirical measurements, etc.

To summarize, the analysis based on spatial distributions is more general, practical and tractable compared to results based on mobility models.

As can be seen later, though in the context of one dimensional case, pursuing \( QoEP_{min} \) and especially \( QoEP_{max} \) is by no means an easy task, even with a single source. As a starting point, our quality of energy provisioning problems, with respect to single source and multiple sources respectively, can be defined as follows.
Quality of Energy Provisioning Problem for Single Source (Multiple Sources): Assume that there is a single source (multiple sources) and a set of nodes in a one dimensional region $\Omega$. Given nodes’ spatial distribution $f_{\text{dis}}(x)$, maximum speed $v_{\text{max}}$ and battery capacity $E_{\pi}$, the quality of energy provisioning problem for single source (multiple sources) is to determine $QoEP_{\text{min}}$ and $QoEP_{\text{max}}$ since there are potentially multiple mobility models obey the same spatial distribution $f_{\text{dis}}(x)$.

We emphasize that our work is not confined to 1D case, as many useful results are also found in 2D and 3D cases.

III. SINGLE SOURCE CASE

In this section, we investigate the QoEP in one dimensional case with a single source, as a preliminary study.

![Illustration of One Dimensional Movement](image1)

![Illustration of Region Partition](image2)

Figure 1. Illustrations of Single Source in One Dimensional Case

Assume that the source is placed at the origin while the node’s movement is confined to a finite line segment $[-x_m, x_m]$. For simplicity, suppose the node spatial distribution $f_{\text{dis}}(x)$ is symmetric, namely $f_{\text{dis}}(x) = f_{\text{dis}}(-x)$ for any $x \in [-x_m, x_m]$. Then we are able to obtain exact QoEPs in some special cases.

Theorem 3.1: Given a source placed at the origin, a node moves on a line $[-x_m, x_m]$ according to some mobility model which results in a node spatial distribution $f_{\text{dis}}(x)$. Then $QoEP = 2 \int_{x_{T}}^{x_m} p_r(x) f_{\text{dis}}(x)dx$ for $p_r(0) < p_s$, and $QoEP = 1$ for $p_r(x_{T}) \geq p_s$.

Proof: If $p_r(x_{T}) \geq p_s$, then a node can sustain normal working anywhere and anytime. Hence $QoEP = \int_{\Omega} LQoEP(x) f_{\text{dis}}(x) dx = 1$ according to (4). If $p_r(0) < p_s$, it indicates that any location in the area $\Omega$ cannot provide sufficient recharge power for normal working. Then the node shall exhaust all its initial energy in a finite period of time, which can be ignored in a normal long period. Then $QoEP = \int_{\Omega} LQoEP(x) f_{\text{dis}}(x) dx = 2 \int_{x_{T}}^{x_m} p_r(x) f_{\text{dis}}(x)dx$. The result follows.

Except for these two special cases, we can only estimate the lower and upper bounds of QoEP.

A. Lower Bound Analysis

As Figure 1 shows, the whole area is divided into two regions, $\Omega_i$ and $\Omega_o$ by $x_T$ and $-x_T$, such that nodes in region $\Omega_i$ are guaranteed to receive a power no less than than $p_s$, while that in $\Omega_o$ are not. Hence we have $p_r(x_{T}) = p_s$, and $x_T = \sqrt{\frac{Z}{p_s}} - \beta$.

Theorem 3.2: Under the condition that $p_r(0) \geq p_s$ and $p_r(x_{T}) < p_s$, the lower bound of QoEP with single source in one dimensional case is:

$$QoEP_{\text{min}} = 2 \left( \int_{x_{T}}^{x_m} f_{\text{dis}}(x) dx + \int_{x_{T}}^{x_m} f_{\text{dis}}(x) \frac{p_r(x)}{p_s} dx \right)$$

and it is tight.

Proof: First, given a mobility model $\mathcal{M}_1$, we can construct a new one $\mathcal{M}_1^*$ by slowing down the speed at any time with a constant factor $c$. Apparently $\mathcal{M}_1^*$ obeys the same spatial distribution, $f_{\text{dis}}(x)$, followed by $\mathcal{M}_1$. As illustrated in Figure 1(b), for any node starting from $x_T$ to region $\Omega_o$, following mobility model $\mathcal{M}_1^*$, $x_Z$ is the furthest location it can reach with nonzero residual energy. This situation occurs only when the node starts with a fully charged energy state, and maintains a constant speed $cv_{\text{max}}$ before it reaches $x_Z$. Hence we have:

$$E_{\pi} - \int_{x_{T}}^{x_Z} \left( \frac{p_s - p_r(x)}{cv_{\text{max}}} \right) dx = 0$$

Then we can calculate the QoEP of $\mathcal{M}_1^*$ as follows:

$$QoEP = \int_{\Omega} LQoEP(x) f_{\text{dis}}(x) dx$$

$$= 2 \left( \int_{x_{T}}^{x_m} f_{\text{dis}}(x) dx + \int_{x_{T}}^{x_m} LQoEP(x) f_{\text{dis}}(x) dx \right)$$

$$+ \int_{x_{T}}^{x_m} f_{\text{dis}}(x) \frac{p_r(x)}{p_s} dx$$

$$\geq 2 \left( \int_{x_{T}}^{x_m} f_{\text{dis}}(x) dx + \int_{x_{T}}^{x_m} f_{\text{dis}}(x) \frac{p_r(x)}{p_s} dx \right)$$

Note that $LQoEP(x) \geq \frac{p_r(x)}{p_s}$ for $x \in [x_T, x_Z]$ due to the potential nonzero residual energy of the node. Then the right side of inequality (8) is indeed a lower bound. Next we continue to prove its tightness.

Given an arbitrarily small value $\varepsilon$.

$$QoEP - QoEP_{\text{min}} = 2 \int_{x_{T}}^{x_m} \left( LQoEP(x) - \frac{p_r(x)}{p_s} \right) f_{\text{dis}}(x) dx$$

$$\leq 2 \int_{x_{T}}^{x_m} 1 \cdot f_{\text{dis}}(x) dx$$

$$\leq 2c_1(x_Z - x_T)$$

where $c_1 = \max_{x \in \Omega} f_{\text{dis}}(x)$. Let $x_Z = \min\{x_m, x_T + \varepsilon/2c_1\}$, then $2c_1(x_Z - x_T) \leq \varepsilon$. Then we substitute it into Equation (6) and obtain:

$$c = \frac{1}{E_{\pi} \varepsilon v_{\text{max}}} \int_{x_{T}}^{\min\{x_m, x_T + \varepsilon/2c_1\}} (p_s - p_r(x)) dx$$

Apparently, we have $QoEP - QoEP_{\text{min}} \leq \varepsilon$ for $c$ given by (9). Hence the tightness of the lower bound is proved. The result follows.
B. Upper Bound Analysis

Due to symmetry of the topology, we only need to consider subregion \([0, x_m]\), and still use \(\Omega_i\) and \(\Omega_o\) to denote the area \([0, x_T]\) and \([x_T, x_m]\) respectively as long as no confusion can arise. Next we’ll first propose some new conceptions to facilitate our further study. Then we launch an investigation about the optimal mobility model through our pattern analysis technique. Based on the gained information, the upper bound can be computed by an effective algorithm.

B.1 Related Conceptions

We introduce the related conceptions as follows.

First of all, we associate location \(x \in \Omega_o\) with the expected battery energy consumption rate, i.e., the average expected battery energy consumption rate \(x\Omega\) the upper bound can be computed by an effective algorithm.

**Theorem 3.3**

Let \(\Lambda_c(\Omega_o) = 0\), which means \(\Lambda_c(x) = 0\) for any \(x \in \Omega_o\), a node keeps an empty battery after it enters region \(\Omega_o\). This situation is the same as that we considered in proof of Theorem 3.2, and therefore we conclude that the QoEP in this case is exactly equal to \(QoEP_{min}\).

Now we consider the case where \(\Lambda_c(\Omega_o) > 0\). As no energy charging loss happens in region \(\Omega_o\), we are able to view the process of energy consumption in nodal capacity as independent from that of the consumption of received energy. Due to the linear additivity of energy consumption, \(\Lambda_c(\Omega_o)\) eventually transforms into a plus to the original QoEP, which is given by: \(\lim_{t \to \infty} \frac{2\Lambda_c(\Omega_o)(t-t_0)}{p_s(t-t_0)} = 2\Lambda_c(\Omega_o)\).

Combining Equation (5), the theorem is proved.

**Lemma 3.3**

For any location \(x \in \Omega_o\), its energy providing ability \(p_{ep}(x)\) is subject to:

\[
p_{ep}(x) \leq \max\{0, \frac{1}{2} f_{dis}(x) v_{\text{max}} (E_p - \int_{y=x}^{x} \frac{p_s - p_r(y)}{v_{\text{max}}} dy)\}
\]

**Proof**: We discuss the issue from two aspects.

First of all, the time a node stays in the adjacent infinitesimal area \(\delta x\) near \(x\) during \([t_0, t]\) is defined as \(T_{\delta x}(t)\). According to the definition of node spatial distribution, we know \(T_{\delta x}(t) = f_{dis}(x) \delta x (t-t_0)\) when \(t \to \infty\). Clearly, a node appears in area \(\delta x\) during time interval \([t_0, t]\) can only choose to travel across the area or stay within it. Let the times of crossing during \([t_0, t]\) be \(2M(t)\) according to (15), and the speed of the node for \(i_{th}\) crossing be \(v_i(x)\). Then we obtain \(T_{\delta x}(t) = \sum_{i=1}^{M(t)} \frac{\delta x}{v_i} + \tau_{\delta x}(t)\) where \(\tau_{\delta x}(t)\) corresponds to the cumulative staying time of the node. Apparently, if \(v_i = v_{\text{max}}\) and \(\tau_{\delta x}(t) = 0\), \(M(t)\) will be maximized. Hence:

\[
M(t) \leq \frac{1}{2} T_{\delta x}(t) v_{\text{max}} / \delta x
\]

\[
\leq \frac{1}{2} f_{dis}(x) v_{\text{max}} (t-t_0)
\]

**B.2 Pattern Analysis**

Based on the definition of node spatial distribution, we can introduce the related conceptions as follows.

Similarly, the expected battery energy harvest rate for location \(x \in \Omega_i\) is defined by:

\[
\lambda_h(x) = \lim_{t \to \infty} \frac{t_2(x)}{t-t_0} = \lim_{t \to \infty} \frac{t(x) - t_2(x)}{t-t_0} (p_r(x) - p_s)
\]

\[
\leq f_{dis}(x) (p_r(x) - p_s) = \lambda_h^{\text{max}}(x)
\]

where \(t_2(x)\) is the cumulative time the node stays at location \(x\) when its residual energy is 0. Accordingly, the expected battery energy consumption rate \(\Lambda_c(\Omega_i)\) is subject to:

\[
\Lambda_c(\Omega_i) = \int_\Omega \lambda_c(x) dx \leq \int_\Omega \lambda_{c, \text{max}}(x) dx = \int_{\Omega'} \lambda_{c, \text{max}}(x) dx
\]

**Lemma 3.4**

The expected battery energy consumption rate should be constant in the long run, which means \(\lim_{t \to \infty} \Lambda_c(\Omega_i)(t-t_0) = \lim_{t \to \infty} \Lambda_h(\Omega_i)(t-t_0)\). The result follows.
Secondly, the maximum residual energy of node upon its traveling across location \( x \) to region \([x, x_m]\) is given by \( \max\{E_{re}^{i,m}(x)\} = \max\{0, E_\pi - \int_{y=x}^{y=x_m} \frac{(p_r - p_v(y))}{\vmax} dy\} \). This situation occurs when node starts with fully recharged energy \( E_\pi \) at \( x \) and maintains constant speed \( v_{max} \) along the path \( x \rightarrow x \). On the other hand, the minimum residual energy of node upon its traveling across location \( x \) to region \([0, x] \) is 0.

Hence:

\[
p_{re}(x) \leq \lim_{t \to \infty} \frac{M(t)}{1 - t_0} \left( \max\{E_{re}^{i,m}(x)\} - \min\{E_{re}^{o,m}(x)\} \right)
\leq \max\{0, \frac{1}{2} f_{dis}(x) v_{max} (E_\pi - 2 \int_{y=x_T}^x \frac{(p_s - p_r(y))}{\vmax} dy) \} \quad (18)
\]

For convenience, we refer to the maximum value of \( p_{re}(x) \) as \( p_{re}^{max}(x) \). The proof of Lemma 3.3 reveals that the faster a node moves, the larger \( p_{re}(x) \) it may gain. Actually, the faster speed accelerates the energy exchange between region \( \Omega_i \) and \( \Omega_o \), and finally leads to an improvement of QoEP.

According to the analysis of the effective energy providing ability in Lemma 3.3, we have a lemma as follows:

**Lemma 3.4:** For any \( x \in \Omega_o \) (\( x_T \leq x < x_m \)),

\[
\Lambda_c(\Omega_o) = \Lambda_c(\Omega_o - \Omega^{x_T}) + p_{re}(x) \quad (19)
\]

and:

\[
\Lambda_c(\Omega_o) \leq \Lambda_c^{max}(\Omega_o - \Omega^{x_T}) + \hat{p}^{max}_{re}(x) \quad (20)
\]

where:

\[
\hat{p}^{max}_{re}(x) = \max\{0, \frac{1}{2} f_{dis}(x) v_{max} (E_\pi - 2 \int_{y=x_T}^x \frac{(p_s - p_r(y))}{\vmax} dy) \} \quad (21)
\]

**Proof:** According to Lemma 3.2, we have \( \Lambda_c(\Omega_o) = \Lambda_c(\Omega_o - \Omega^{x_T}) + \Lambda_c(\Omega^{x_T}) = \Lambda_c(\Omega_o - \Omega^{x_T}) + p_{re}(x) \), the first equality is straightforward.

Next, recall the proof of Lemma 3.3, we know that if \( t \to \infty \), there are at most \( M(t) = \frac{1}{2} f_{dis}(x) v_{max} (t - t_0) \) trips of node into region \([x, x_m]\) in time period \([t_0, t]\). Now we are concerned with energy brought into region \([x, x_m]\) for each trip such that the overall \( \Lambda_c(\Omega_o) \) is maximized.

We first consider the case that \( \Lambda_c(\Omega_o - \Omega^{x_T}) \) is maximized, that is, when a node travels into region \([x_T, x]\), it always has nonzero battery energy. As we mentioned before, the maximum residual energy of node upon its traveling across location \( x \) to region \([x, x_m]\) is \( \max\{E_{re}^{i,m}(x)\} = E_\pi - \int_{y=x}^{y=x_m} \frac{(p_r - p_v(y))}{\vmax} dy \). In addition, when the node travels across location \( x \) to \([x_T, x]\), it can either linger in area \([x_T, x]\) for some time, then enter into region \([x, x_m]\) again; or move into region \( \Omega_i \), namely \([0, x_T]\), at last, before its next entrance into \([x, x_m]\). For the latter case, the minimum required energy for the node at \( x \) to sustain normal operation from \( x \) until it arrives at \( x_T \) is determined by \( \min\{E_{re}^{i,m}(x)\} = \int_{y=x}^{y=x_T} \frac{(p_r - p_v(y))}{\vmax} dy \), which is corresponding to the situation that the node maintains the maximum speed \( v_{max} \). As for the former case, note that node is doomed to move back into \( \Omega_i \) again after some period of time, when its battery energy is required to be no less than \( \min\{E_{re}^{i,m}(x)\} \) at \( x \), and the truth that the battery energy of node keeps decreasing when it moves within \( \Omega_o \). Thus node battery energy should be higher than \( \min\{E_{re}^{i,m}(x)\} \) in this case, and the energy that the node brings into the area further in one trip is no more than \( \max\{0, E_\pi - 2 \int_{y=x_T}^x \frac{(p_r - p_v(y))}{\vmax} dy\} \). Sum up, we have:

\[
p_{re}(x) \leq \lim_{t \to \infty} \frac{M(t)}{1 - t_0} \left( \max\{E_{re}^{i,m}(x)\} - \min\{E_{re}^{o,m}(x)\} \right)
\leq \max\{0, \frac{1}{2} f_{dis}(x) v_{max} (E_\pi - 2 \int_{y=x_T}^x \frac{(p_s - p_r(y))}{\vmax} dy) \}
\leq \hat{p}^{max}_{re}(x)
\]

then:

\[
\Lambda_c(\Omega_o) = \Lambda_c(\Omega_o - \Omega^{x_T}) + p_{re}(x)
\leq \Lambda_c^{max}(\Omega_o - \Omega^{x_T}) + \hat{p}^{max}_{re}(x)
\]

Secondly, we proceed to consider the case if \( \Lambda_c(\Omega_o - \Omega^{x_T}) \) is not maximized. Suppose the expected battery energy consumption rate for region \( \Omega_o - \Omega^{x_T} \) is \( \Lambda_c^1(\Omega_o - \Omega^{x_T}) \) \( \Lambda_c^1(\Omega_o - \Omega^{x_T}) \leq \Lambda_c^{max}(\Omega_o - \Omega^{x_T}) \). In this case, \( E_{re}^{i,m}(x) \leq \max\{0, E_\pi - \int_{y=x_T}^x \frac{(p_r - p_v(y))}{\vmax} dy\} \) still holds.

Therefore, if \( E_{re}^{i,m}(x) \geq \int_{y=x_T}^x \frac{(p_r - p_v(y))}{\vmax} dy \) for \( m = 1, \ldots, M \), we still have:

\[
\Lambda_c(\Omega_o) = \Lambda_c^1(\Omega_o - \Omega^{x_T}) + \lim_{t \to \infty} \frac{1}{t - t_0} (\frac{\Sigma M(t)}{t_0} (E_{re}^{i,m}(x) - E_{re}^{o,m}(x)))
\leq \Lambda_c^1(\Omega_o - \Omega^{x_T}) + \lim_{t \to \infty} \frac{M(t)}{t - t_0} (\max\{E_{re}^{i,m}(x)\} - \min\{E_{re}^{o,m}(x)\})
\leq \Lambda_c^{max}(\Omega_o - \Omega^{x_T}) + \hat{p}^{max}_{re}(x)
\]

Otherwise, suppose that \( E_{re}^{i,m}(x) < \int_{y=x_T}^x \frac{(p_r - p_v(y))}{\vmax} dy \) for all \( m = 1, \ldots, M \) in the worst case, then we perform the following adjustments. We artificially shorten the time the node stays in \([x, x_m]\) such that its battery energy accordingly increases to \( \int_{y=x_T}^x \frac{(p_r - p_v(y))}{\vmax} dy \) when it travels across \( x \), namely \( E_{re}^{i,m}(x) = \int_{y=x_T}^x \frac{(p_r - p_v(y))}{\vmax} dy \) (if \( E_{re}^{i,m}(x) < \int_{y=x_T}^x \frac{(p_r - p_v(y))}{\vmax} dy \), which make it possible for such adjustment, we then set \( E_{re}^{i,m}(x) = \int_{y=x_T}^x \frac{(p_r - p_v(y))}{\vmax} dy \)). In other words, there is an amount of surplus energy \( \Delta E_{re}^{i,m} = \int_{y=x_T}^x \frac{(p_r - p_v(y))}{\vmax} dy - E_{re}^{i,m}(x) \) added to the node battery upon its \( m_{th} \) crossing from region \([x, x_m]\) to \([x_T, x]\). We claim that \( \Delta E_{re}^{i,m} \) will be entirely
consumed in $\Omega_o$, since $E_{re}^{o,m}(x) = \int_{y=y_{T}}^{x} \frac{(p_{r} - p_{s})}{v_{max}} dy$ can guarantee that no matter how the node moves, it must exhaust all its battery energy when it comes into region $\Omega_i$. Moreover, it is obvious that the newly expected battery energy consumption rate for region $\Omega_o - \Omega^{\geq x}$ is still no more than $\Lambda_c^{max}(\Omega_o - \Omega^{\geq x})$. Then we have:

$$\Lambda_c(\Omega_o) = \Lambda_c^1(\Omega_o - \Omega^{\geq x}) + \lim_{t \to \infty} \frac{1}{t - t_0} (\Sigma_{m=1}^{M(t)} (E_{re}^{i,m}(x) - E_{re}^{i,m}(x)))$$

$$\leq \lim_{t \to \infty} \frac{1}{t - t_0} \Lambda_c^1(\Omega_o - \Omega^{\geq x}) + \lim_{t \to \infty} \frac{1}{t - t_0} (\max \{E_{re}^{i,m}(x)\} - \int_{y=y_{T}}^{x} \frac{(p_{s} - p_{r}(y))}{v_{max}} dy)$$

$$\leq \Lambda_c^{max}(\Omega_o - \Omega^{\geq x}) + \lim_{t \to \infty} \frac{M(t)}{t - t_0} (\max \{E_{re}^{i,m}(x)\} - \int_{y=y_{T}}^{x} \frac{(p_{s} - p_{r}(y))}{v_{max}} dy)$$

$$= \Lambda_c^{max}(\Omega_o - \Omega^{\geq x}) + \frac{\Lambda_{ef}^{max}(x)}{v_{max}}$$

(22)

In addition, if $E_{\pi} < 2 \int_{y=y_{T}}^{x} \frac{(p_{s} - p_{r}(y))}{v_{max}} dy$, we can easily get $\Lambda_c(\Omega_o) \leq \Lambda_c^{max}(\Omega_o - \Omega^{\geq x})$, the result follows.

Figure 2. Illustration of Flow Pattern

Let $x = y_{T}$ in Lemma 3.2 and Lemma 3.3, and combine them with Theorem 3.2, 3.3, we can obtain a relaxed upper bound of QoEP.

Corollary 3.1: Under the condition that $p_{r}(0) \geq p_{s}$ and $p_{r}(x_{m}) < p_{s}$, QoEP is subject to:

$$QoEP \leq QoEP_{min} + \frac{f_{dis}(x_{T}) v_{max} E_{\pi}}{p_{s}}$$

(23)

The right side of the inequality can be used as estimation for upper bound of QoEP in some scenarios, due to its simplicity. As for tight upper bound, we have to figure out how to maximize $\Lambda_c(\Omega_o)$ according to Theorem 3.3. In the next section, we employ a novel technique to address this issue.

B.2 Flow Pattern and Optimal Mobility Model

We treat the energy providing ability problem by making an analogy between node’s movement and flow.

1) Introduction of Flow Pattern: As illustrated in Figure 2, three flows contribute to the final spatial distribution $f_{dis}(x)$, we denote them I, II, and III respectively. The flow in area I is stationary, which is corresponding to the situation where node pauses at location $x$. The outflow $\Gamma_+$ and inflow $\Gamma_-$ reflect the accumulative tendency for node to move away from or towards the source at location $x$ respectively. Similar to the corresponding conceptions in physics, we have the following definitions:

- **flux**: The flux of $\Gamma$, denoted by $F(\Gamma, x)$, is defined as the frequency of node move towards the reference direction at location $x$:

$$F(\Gamma, x) = \lim_{t \to \infty} \frac{M(t)}{t - t_0}$$

(24)

where $M(t)$ is the crossing times of node at location $x$ along the flow’s reference direction during time interval $[t_0, t]$.

- **speed**: It is denoted by $V(\Gamma, x)$. As all the flows we discuss later are steady flows, $V(\Gamma, x)$ is indeed a constant rather than a probability distribution of velocity, which is corresponding to that of node at $x$.

- **density**: It is defined by:

$$D(\Gamma, x) = F(\Gamma, x)/V(\Gamma, x).$$

(25)

Actually it is corresponding to the probability of node with velocity $V(\Gamma, x)$ at $x$.

- **power**: It is defined by:

$$P(\Gamma, x) = D(\Gamma, x)E_{re}(\Gamma, x)$$

(26)

where $E_{re}(\Gamma, x)$ denotes the nodal residual energy at $x$ in flow $\Gamma$.

Extending this analogy to the whole area, we find that there are infinite flows distributed anywhere, and the oppositely directed flows at any location must strike a balance with the net flux equals to 0. We name such distribution of flows corresponding to a mobility model as flow pattern and denote it as $F$. We omit formal definition and detailed analysis of flow pattern here to save space. For analytical convenience, we introduce in the conception of effective flow $\Gamma_e$, as shown in Figure 2, to represent the combination of inflow and outflow. Note that the reference direction of $\Gamma_e$ is identical with that of its positive flow component $\Gamma_+$ and opposite to negative flow component $\Gamma_-$. Further we define its related parameters as follows:

- **effective flux**: $F_e(\Gamma_e, x) = F(\Gamma_+, x) + F(\Gamma_-, x)$.

- **effective speed**: $V_e(\Gamma_e, x) = V(\Gamma_+, x) - V(\Gamma_-, x)$.

- **effective density**: $D_e(\Gamma_e, x) = D(\Gamma_+, x) + D(\Gamma_-, x)$.

- **effective power**: $P_e(\Gamma_e, x) = P(\Gamma_+, x) - P(\Gamma_-, x)$.

Combining (17), (24), we have:

$$F(\Gamma_+, x) = F(\Gamma_-, x) \leq \frac{1}{2} f_{dis}(x) v_{max}$$

(27)
and:

\[ F_e(Γ_e, x) ≤ F_{e, max}(Γ_e, x) = f_{dis}(x)v^{max} \]  \( (28) \)

The intention of analysis on flow pattern is to grasp the features of optimal mobility model which achieves the largest QoEP. As can be seen later, these features also guide us to design optimal mobility model.

2) Flow Pattern Analysis of Optimal Mobility Model: According to Theorem 3.3, to maximize QoEP we have to maximize \( Λ_i(Ω_i) \), which is also equal to \( Λ_i(Ω_i) \) as Lemma 3.1. Hence, we propose the following assumption conditions to decouple the problem and analyze \( Λ_i(Ω_i) \) and \( Λ_h(Ω_i) \) separately: upon node moving across location \( x_T \), (i) its speed equals \( v^{max} \); and (ii) it either depletes all its energy as it returns back from \( Ω_o \), or is fully recharged as returns from region \( Ω_i \). Then, suppose there are two effective flows, \( Γ_m \) and \( Γ'_m \), entering into \( Ω_o \) and \( Ω_i \), respectively, we attempt to derive the sufficient conditions for \( Γ_m \) and \( Γ'_m \) that make the corresponding mobility model optimal. Additionally, we have \( F_e(Γ_m, x_T) = F_e(Γ'_m, x_T) \) since they are actually the same flow except for opposite reference direction.

Next, we’d like to investigate the flow pattern \( 𝒢_o \) in \( Ω_o \). Generally speaking, \( 𝒢_o \) can be divided into two parts: main flow \( Γ_m \) and compensation flow \( Γ_c \).

(a) Main Flow: We refer to \( Γ_m \) as main flow as it is the main object in our following analysis. To convey an understanding of main flow in advance, we outline its key features as follows.

\* Γ_m starts with a largest effective flux at \( x_T \), i.e.

\[ F_e(Γ_m, x_T) = F_{e, max}(Γ_m, x_T) = f_{dis}(x_T)v^{max}. \]

\* Γ_m maintains a constant and largest speed \( V_e(Γ_m, x) = v^{max} \) before its termination. At any location \( x \), the nodal energy in its positive flow component is

\[ E_{r, +}(Γ_m, x) = E_p - \int_{y=x_T}^{x} \frac{(p_r-p_s(y))}{v^{max}(y)} dy, \]

and that in its negative flow component is

\[ E_{r, -}(Γ_m, x) = \int_{y=x_T}^{x} \frac{(p_r-p_s(y))}{v^{max}(y)} dy. \]

\* Before it is terminated, the main flow always extracts subflows from itself, when \( F_e(Γ_m, x) ≤ f_{dis}(x)v^{max} \), so as to ensure the following conditions are met at location \( x \): (i) the spatial distribution is guaranteed; (ii) \( λ_c(x) = \lambda^{max}_c(x) \). Meanwhile, the extracted subflows should be consistent with the main flow.

\* When a so-called flow crash happens, which means that \( F_e(Γ_m, x) > f_{dis}(x)v^{max} \), the assumption conditions will be destroyed. Thus feedback adjustment technique is applied such that the assumption conditions can be re-established. Generally, it finally leads to a suppression of \( Γ_m \).

\* The main flow terminates only when: (a) its power degrades to zero, namely \( F_e(Γ_m, x) = 0 \); and (b) the main flow shrinks down to zero, namely its effective flux \( F_e(Γ_m, x) = 0 \).

Next we describe the last three features in details. Generally, there are potentially two cases that the main flow would encounter after its setting, as shown in Figure 3.

Case I: Suppose the main flow travels from \( x_i \) to \( x_i + δx \) (δx is an infinitesimal value), where \( F_e(Γ_m, x_i + δx) ≤ f_{dis}(x_i + δx)v^{max} \), which means that \( D_e(Γ_m, x_i + δx) ≤ f_{dis}(x_i + δx) \). To guarantee the spatial distribution at location \( x_i + δx \), \( Γ_m \) should extract a sub-flow \( δΓ_m \) from itself into region \([x_i, x_i + δx]\). Specifically, the node in this sub-flow will first travel to location \( x_i + δx \) and then turn back to \( x_i \), with a constant speed \( V_e(δΓ_m, x_i) \). Finally it integrates into flow \( Γ_m \) again. Hence:

\[ -δF_e(Γ_m, x_i) + F_e(Γ_m, x_i + δx) = f_{dis}(x_i + δx) \]  \( (29) \)

Note that \( F_e(δΓ_m, x_i) = −δF_e(Γ_m, x_i) \). Meanwhile, during the time within the area \([x_i, x_i + δx]\), the node should consume exactly an amount of energy

\[ E_p - 2 \int_{x=x_T}^{x_i} \frac{(p_r-p_s(x))}{v^{max}(x)} dx \]  \( \text{for two reasons: (i)} \)

(i) when node turns back to \( Γ_m \), its left energy \( E_{r, -}(δΓ_m, x_i) = E_{r, +}(Γ_m, x_i) - (E_p - 2 \int_{x=x_T}^{x} \frac{(p_r-p_s(x))}{v^{max}(x)} dx) = \int_{x=x_T}^{x_i} \frac{(p_r-p_s(x))}{v^{max}(x)} dx \) \( \text{is } E_{r, -}(Γ_m, x_i), \) which is consistent with the main flow. (ii) since node in \( Γ_m \) and \( δΓ_m \) both have nonzero energy, we have \( λ_c(x) = \lambda^{max}_c(x) \) for \( x \in [x_i, x_i + δx] \) according to the definition of \( λ_c(x) \).

Hence:

\[ (p_r-p_s(x)) \frac{2δx}{V_e(δΓ_m, x_i)} = E_p - 2 \int_{x=x_T}^{x_i} \frac{(p_r-p_s(x))}{v^{max}(x)} dx \]  \( (30) \)

Combining (29) and (30), and transform the results into the form of differential equations:

\[ dF_e(Γ_m, x) = \frac{-2(f_{dis}(x) - F_e(Γ_m, x)/v^{max}) (p_r-p_s(x))}{(E_p - 2 \int_{y=x_T}^{x} \frac{(p_r-p_s(y))}{v^{max}(y)} dy)v^{max}} dx \]  \( (31) \)

\[ V_e(Γ_m, x) = \frac{2(p_r-p_s(x))}{E_p - 2 \int_{y=x_T}^{x} \frac{(p_r-p_s(y))}{v^{max}(y)} dy} \]  \( (32) \)

for \( F_e(Γ_m, x) ≤ f_{dis}(x)v^{max} \).

Case II: Suppose the main flow travels from \( x_j \) to \( x_j + δx \), where \( F_e(Γ_m, x_j + δx) > f_{dis}(x_j + δx)v^{max} \) (or \( D_e(Γ_m, x_j + δx) > f_{dis}(x_j + δx) \)). In this case, a so-called flow crash happens, as is shown in Figure 3, with a portion
of the flow turning back immediately, and $F_e(\Gamma_m, x_j + \delta x)$ being suppressed to $f_{dis}(x_j + \delta x)v^{max}$. Even worse, the returned node must have nonzero residual energy when it arrives at $x_T$, which destroys the assumption conditions.

To address this problem, we employ a feedback adjustment technique which resembles to some extent the back pressure congestion control in networks. Taking Figure 4 as an example, the main flow $\Gamma_m^{(1)}$ starts from $x_T$, and gradually shrinks until it reaches $x_T^1$, where a flow crash occurs. Such flow crash constantly happens before minimum point $x_T^0$ on space distribution curve. The corresponding envelope curve of $\Gamma_m^{(1)}$ is represented by a dotted curve $\gamma^{(1)}$. Then we suppress the initial main flow $\Gamma_m^{(1)}$ to $\Gamma_m^{(2)}$, which guarantees no flow crash happens before it reaches $x_T^1$, as curve $\gamma^{(2)}$ depicts, and $F_e(\Gamma_m^{(2)}, x_T^1) = f_{dis}(x_T^1)v^{max}$ at $x_T^1$. Likewise, $\Gamma_m^{(2)}$ can be further reduced to $\Gamma_m^{(3)}$, and $\gamma^{(2)}$ falls to be $\gamma^{(3)}$ when another flow crash occurs before minimum point $x_T^2$. Besides, this technique can also be applied to handle the flow crash happens right before $x_m$. For simplicity, we still use $\Gamma_m$ to denote the effective flow after its last feedback adjustment.

There are two cases for termination of the main flow: (a) its power degrades to zero, namely $P_e(\Gamma_m, x_T) = 0$, at some location $x_T^p$ as $\gamma^{(4)}$ shows. Note that the main flow doesn’t necessarily shrink down to zero in this case. We can further derive that $x_T^p$ is given by $E_{\text{eq}} - 2 \int_{x_T^p}^{x_T} \left( \frac{p_{s} - p_{s}(x)}{\theta(x)} \right) dx = 0$, and (b) the main flow shrinks down to zero at some location $x_T^f$ before $x_T^p$, namely $F_e(\Gamma_m, x_T^f) = 0$, as curve $\gamma^{(5)}$ shows. We name these two cases as zero power ending and zero flux ending respectively.

(b) Compensation Flow: Suppose $\Gamma_m$ terminates at $x_T^f$ ($x_T^f = x_T^f$ or $x_T^f$). To guarantee the spatial distribution of rest areas, we need to initiate an so-called compensation flow $\Gamma_c$ at $x_T^f$. We set its effective flux $F_c(\Gamma_c, x_T^f) = f_{dis}(x_T^f)v^{max}$ and effective power $P_c(\Gamma_c, x_T^f) = 0$. For its effective speed, we adopt a uniform flow scheme, i.e., its speed $V_c(\Gamma_c, x) = f_{dis}(x)V_c(\Gamma_c, x)$. Then the spatial distribution of the left areas will be guaranteed. Moreover, we don’t need to care about flow crash in this situation for node in the flow has no energy throughout the process.

Generally, the analysis on the flow pattern $\mathcal{F}$ is very similar to that of $\mathcal{F}_c$. For example, it also consists of main flow $\Gamma_m$, and compensation flow $\Gamma_c$. $\Gamma_m$ also maintains a constant speed $v^{max}$ as $\Gamma_m$ does. More importantly, $\lambda_h(x) = \lambda^{max}_h(x)$ is guaranteed at any location $x$ before its termination. More detailed descriptions are omitted to save space.

Now we attempt to construct the overall flow pattern $\mathcal{F}$ based on $\mathcal{F}_c$ and $\mathcal{F}_i$. As is stated above, assumption conditions require that $F_e(\Gamma_m, x_T) = F_e(\Gamma_m^*, x_T)$. For this reason, if $F_e(\Gamma_m, x_T) > F_e(\Gamma_m^*, x_T)$, without loss of generality, we suppress the main flow $\Gamma_m$ at $x_T$, like feedback adjustment technique does, until $F_e(\Gamma_m, x_T)$ equals $F_e(\Gamma_m^*, x_T)$. We denote the revised effective flows as $\Gamma_m$ and $\Gamma_m^*$ respectively. $\mathcal{F}$ is therefore composed by $\Gamma_m, \Gamma_c, \Gamma_m^*$ and $\Gamma_m^*$.

3) Implementation of Optimal Mobility Model: Based on the flow pattern $\mathcal{F}$ we obtained, we are concerned with its implementation into mobility model in this section.

Generally speaking, we repeatedly implement each flow in rounds. In each round, we first implement main flows of $\mathcal{F}$, namely $\Gamma_m$ and $\Gamma_m^*$, simultaneously, then $\Gamma_c$ and $\Gamma_c^*$ respectively.

For $\Gamma_m$, we divide region $[x_T, x_M]$ uniformly into $P (P \in \mathbb{N}^+, P \rightarrow \infty)$ subregions, and let the node spend trips into region $\Omega_m$, with starting and end point both being at $x_T$. In particular, there are $q_k = \frac{\delta F_e(\Gamma_m, x_T)}{F_e(\Gamma_m, x_T)}kQ - \sum_{j=1}^{k-1} q_j$ ($Q \in \mathbb{N}$ is a predefined value) trips launched for subregion $[x_i, x_i + \delta x]$ in round $k$. Moreover, as is stated above, node sustains a speed of $V_e(\Delta x)$ within $[x_i, x_i + \delta x]$ as subflow $\delta \Gamma_m$ does, while keeps a constant speed $v^{max}$ outside.

Meanwhile, the node also spends $Q$ trips into region $\Omega_m$ to implement $\Gamma_m^*$, the distribution of trips and node’s speed are also determined by referring to the effective flux and speed of $\Gamma_m$ and its subflows. We note that implementations of $\Gamma_m$ and $\Gamma_m^*$ are interleaved with each other, as each trip for $\Omega_m$ is initiated right after the end of an trip launched into $\Omega_m$, and vice versa.

We then implement $\Gamma_c$ and $\Gamma_c^*$ in a similar way. After this, we go back to implement $\Gamma_m$ and $\Gamma_m^*$ again to start a new round. The process will be repeated forever.

Actually, this implementation solution can be further refined by adopting uniform flow scheme. Taking $\Gamma_m$ as an example, suppose that we’ve obtained the precise value of $F_e(\Gamma_m, x)$ by solving differential equation (31) for $\Gamma_m$, then we can substitute all the subflows in some region $[x, x + l]$ with a single uniform subflow $\Gamma_l$ which satisfies: $F_e(\Gamma_l, x) = F_e(\Gamma_m, x) - F_e(\Gamma_m, x + l)$ for any $x_i \in [x, x + l]$ ($l$ is non-infinitesimal). Due to energy conservation nature, this uniform flow is consistent with the main flow, since its injected energy into region $[x, x + l]$ is the same as the sum of those for subflows because $\lambda_{\text{eq}}(x_i) = \lambda^{max}_{\text{eq}}(x_i)$ for any $x_i \in [x, x + l]$ are guaranteed for both cases. However, we need to ensure that the effective speed at any $x_i \in [x, x + l]$ of this uniform subflow does not exceeds $v^{max}$. Therefore
the length of subregion should be carefully designed. It is obvious that as long as $V_c(d\Gamma_m, x_i) \leq \mu_{\text{max}}$, we are always able to find such a non-infinitesimal $l$. However, if $E_\pi \rightarrow 2 \int_{y=x_p}^x \frac{(p(x) - p(y))}{\mu_{\text{max}}} dy$ (or $x \rightarrow x_{zp}$), $V_c(d\Gamma_m, x_i)$ will become considerable high. To solve this problem, we can similarly substitute the subflows in $[x_{zp} - l_i, x_{zp}]$ with uniform flow with an appropriate $l_i$.

By doing so, we divide the active areas of $\Gamma_m$ into $M$ finite subregions $\Upsilon_i, i = \{1, 2, 3, \ldots, M\}$ in the sequence of distance to source ($\Upsilon_j$ is further than $\Upsilon_i$ for $j > i$). Denote by $\Gamma_{\Upsilon_i}$ their corresponding uniform subflow. Apparently, there are $M - i + 1$ subflows, namely $\Gamma_{\Upsilon_1}, \Gamma_{\Upsilon_2}, \ldots, \Gamma_{\Upsilon_M}$ at subregion $\Upsilon_i$. Similarly, there are $q_k = \frac{F_c(\Gamma_{\Upsilon_j}, x_j)}{F_c(\Gamma_{\Upsilon_M}, x_M)} kQ - \sum_{j=1}^{k-1} q_j$ trips launched to implement $\Gamma_{\Upsilon_j}$ at round $k$.

Next we are going to prove our implemented mobility model achieves spatial distribution $f_{\text{dis}}(x)$ and theoretical $\text{QoEP}$ by flow pattern analysis.

Clearly, at round $N$ there are totally $\frac{F_c(\Gamma_{\Upsilon_j}, x_j)}{F_c(\Gamma_{\Upsilon_M}, x_M)} NQ$ trips launched into subregion $\Upsilon_j$ to implement $\Gamma_{\Upsilon_j}$. As a result, the difference between the current node spatial distribution $f_{\text{dis}}^N(x_i)$ ($x_i \in \Upsilon_i$) at round $N$ and the original node spatial distribution $f_{\text{dis}}(x_i)$ is given by:

$$|\Delta f_{\text{dis}}(x_i)| = |f_{\text{dis}}^N(x_i) - f_{\text{dis}}(x_i)|$$

$$\leq \sum_{j=1}^{M} \frac{|F_c(\Gamma_{\Upsilon_j}, x_j)|}{F_c(\Gamma_{\Upsilon_M}, x_M)} |NQ - F_c(\Gamma_{\Upsilon_j}, x_j)| \cdot D_e(\Gamma_{\Upsilon_j}, x_j)$$

$$\leq \sum_{j=1}^{M} \frac{F_c(\Gamma_{\Upsilon_j}, x_j)}{F_c(\Gamma_{\Upsilon_M}, x_M)} \cdot \frac{1}{NQ}$$

$$= C_{\text{md}} f_{\text{dis}}(x_i)$$

$$\text{(33)}$$

where $C_{\text{md}} = \sum_{j=1}^{M} \frac{F_c(\Gamma_{\Upsilon_M}, x_M)}{F_c(\Gamma_{\Upsilon_j}, x_j)}$ is a constant independent of the location in the active areas of $\Gamma_m$, $D_e(\Gamma_{\Upsilon_j}, x_j)$ is the effective density of $\Upsilon_j$ which satisfies: $\sum_{j=1}^{M} D_e(\Gamma_{\Upsilon_j}, x_j) = 1$. Similarly we can apply the same technique to analyze $\Gamma_m, \Gamma_c$ and $\Gamma_c'$. Finally we obtain:

$$|\Delta f_{\text{dis}}(x)| \leq C_d \frac{1}{NQ} f_{\text{dis}}(x)$$

$$\text{(34)}$$

where $C_d$ is a constant.

Therefore we can define the implementation error of node spatial distribution as:

$$IE_{\text{dis}} = \int_{\Omega} |\Delta f_{\text{dis}}(x)| dx$$

$$\leq \int_{\Omega} \frac{C_d}{NQ} f_{\text{dis}}(x) dx$$

$$= \frac{C_d}{NQ}$$

$$\text{(35)}$$

Moreover, we emphasize that the transient processes between implementations of these flows won’t introduce more errors to $f_{\text{dis}}^N(x)$ or $\text{QoEP}'$. The reason is that these processes are continuous and we can equivalently regard that the battery energy consumption in active areas of $\Gamma_c$ occurs in that of $\Gamma_m$ as their overall battery energy consumptions are identical, as well as the the battery energy harvest in active areas of $\Gamma_c'$. In this sense, the $L\text{QoEP}(x)$ of our implemented mobility model at round $N$ is the same as that of $\mathcal{F}$. Hence we can also define the implementation error of $\text{QoEP}$ as:

$$IE_{\text{QoEP}} = |\text{QoEP}' - \text{QoEP}|$$

$$= |\int_{\Omega} L\text{QoEP}(x) f_{\text{dis}}^N(x) dx - \int_{\Omega} L\text{QoEP}(x) f_{\text{dis}}(x) dx|$$

$$\leq \int_{\Omega} L\text{QoEP}(x) |f_{\text{dis}}^N(x) - f_{\text{dis}}(x)| dx$$

$$\leq \int_{\Omega} |f_{\text{dis}}^N(x) - f_{\text{dis}}(x)| dx$$

$$= \int_{\Omega} |\Delta f_{\text{dis}}(x)| dx$$

$$= IE_{\text{dis}}$$

$$\text{(36)}$$

Consequently, given an arbitrarily small value $\varepsilon$, we set $N \geq \lceil \frac{1}{\varepsilon} \rceil$, then $IE_{\text{dis}} \leq \varepsilon$ as well as $IE_{\text{QoEP}} \leq \varepsilon$. Therefore we conclude that the spatial distribution of our implemented mobility model finally converges to limit $f_{\text{dis}}(x)$, and its $\text{QoEP}$ converges to limit $\text{QoEP}$ of $\mathcal{F}$ given by flow pattern analysis. Finally we denote our implemented mobility model by $\mathcal{M}$.

4) Optimality Analysis and Performance Upper Bound:

First of all, we propose the following lemma.

**Lemma 3.5:** Mobility model $\mathcal{M}$ corresponding to $\mathcal{F}$ is optimal.

**Proof:** First of all, we assume that $F_c(\Gamma_m, x_T) < F_c(\Gamma'_m, x_T)$ in construction of $\mathcal{F}$. Hence $\Gamma_c$ is indeed not suppressed. In this case, assuming that there are $N - 1$ minimum points $x_k^k (k = 1, 2, 3, \ldots, N - 1)$ as shown in Figure 4, and the main flow $\Gamma_m$ terminates at location $x^*$. There are totally three cases as in the following.

Case I: $x^*_m = x^e$. In this case, since $\lambda_c(x) = \lambda_c^{\text{max}}(x)$ for any $x \in \Omega_c$, we have $\Lambda_c(\Omega_c) = \Lambda_c^{\text{max}}(\Omega_c)$. It is undoubtedly maximal.

Case II: $x^*_m = x^z$. For this zero flux ending case, suppose that the last flow crash happens near some minimum point $x_k^z (k = 1, 2, 3, \ldots, N - 1)$ which leads to the last adjustment to the main flow. Clearly, $\Lambda_c(\Omega^z_m - \Omega^z_k) = \Lambda_c^{\text{max}}(\Omega^z_m - \Omega^z_k)$ is maximized as $\lambda_c(x) = \lambda_c^{\text{max}}(x)$ for any $x \in \Omega^z_m - \Omega^z_k$.

We proceed to investigate the energy providing ability at $x_k^z$. Note that $F_c(\Gamma_m, x_k^z) = f_{\text{dis}}(x_k^z)\nu_c^{\text{max}}$ after the last adjustment. According to the definition of the energy providing ability $p_{\text{ep}}(x)$, we have:

$$p_{\text{ep}}(x_k^z) = \lim_{t \to \infty} \frac{1}{t - c_0} (\sum_{m=1}^{N_Q}(E_{\text{re}}^{c,m}(x_k^z) - E_{\text{re}}^{c,m}(x_k^z)))$$
energy consumption rate for region \( \Omega \) after construction of expected battery energy harvest rate for region \( \Omega \) computation becomes easy according to Equation (38).

According to Lemma 3.4, we conclude that \( \Lambda_c(\Omega_o) \) is maximum.

Case III: \( x_{end} = x_{zp} \) \( (x_{zp} < x_m) \). The analysis of zero power ending case is very similar to that of zero flux ending case, except for \( \hat{p}_{ep}(x_g^k) = \hat{p}_{ep}^{max}(x_g^k) = 0 \). Nevertheless, \( \Lambda_c(\Omega_o) \) is still maximum.

After all, according to Theorem 3.3, we conclude that QoEP is maximum in this case.

Likewise, as for \( F_c(\Gamma_m, x_T) > F_c(\Gamma'_m, x_T) \), \( \Gamma'_m \) is not suppressed, we can derive that \( \Lambda_h(\Omega_i) \) is also maximum. According to Lemma 3.1 and Theorem 3.3, we come to the conclusion that QoEP is maximum.

Summing up the analysis above, the result follows.

Then we rewrite \( \mathcal{M} \) as \( \mathcal{M}^{opt} \). Again, we use \( \Lambda_c^{opt}(\Omega_o) \) and \( \Lambda_h^{opt}(\Omega_i) \) to denote the maximum expected battery energy consumption rate for region \( \Omega_o \) and the maximum expected battery energy harvest rate for region \( \Omega_i \). Clearly, after construction of \( \mathcal{R} \), we have \( \Lambda_c(\Omega_o) = \Lambda_h(\Omega_i) = \min\{\Lambda_c^{opt}(\Omega_o), \Lambda_h^{opt}(\Omega_i)\} \).

After all, we have the following theorem.

**Theorem 3.4:** Under the condition that \( p_r(0) \geq p_s \) and \( p_r(x_m) < p_s \), the upper bound of QoEP with single source in one dimensional case is:

\[
QoEP_{max} = QoEP_{min} + 2\min\{\Lambda_c^{opt}(\Omega_o), \Lambda_h^{opt}(\Omega_i)\}
\]

where \( \Lambda_c^{opt}(\Omega_o) \) and \( \Lambda_h^{opt}(\Omega_i) \) are the maximum expected battery energy consumption rate for region \( \Omega_o \) and the maximum expected battery energy harvest rate for region \( \Omega_i \) respectively. It is a tight bound.

**Proof:** According to Lemma (3.5), \( QoEP_{max} \) is undoubtedly an upper bound. In addition, the tightness of \( QoEP_{max} \) is naturally guaranteed by the existence of optimal mobility model \( \mathcal{M}^{opt} \).

\[\text{Algorithm 1 Computation of } \Lambda_c^{max}(\Omega_o)\]

**Input:** the spatial distribution \( f_{dis}(x) \), the maximum speed \( v^{max} \) and the battery capacity of node \( E_r \)

**Output:** the optimal expected battery energy consumption rate of region \( \Omega_o \) \( \Lambda_c^{opt}(\Omega_o) \)

1: Find all minimum points \( x_g^k \) \( (0 \leq k \leq N, x_g^0 = x_T, x_g^N = x_m) \) of \( f_{dis}(x) \), divide \( \Omega_o \) into N sub-regions \( \Omega^k \) accordingly; compute \( x_{zp} \) (s.t. \( E_r - 2 \int_{x=x_T}^{x=x_zp} (p_s - p_r(x))dx = 0 \));

2: \( \Lambda_c(\Omega_o) \) \( \leftarrow 0 \), \( P_e(\Gamma_m, x_g^0) \) \( \leftarrow \hat{p}_{ep}(x_g^0) \{ \text{according to Equation (37)} \} \)

3: \( k \leftarrow 1 \);

4: while \( P_e(\Gamma_m, x_g^{k-1}) > 0 \) and \( k \leq N \) do

5: \( \text{if } x_{zp} \in \Omega^k \) then

6: \( \Lambda_c(\Omega_o) \) \( \leftarrow \Lambda_c(\Omega_o) + \min\{P_e(\Gamma_m, x_g^{k-1}), \Lambda_c^{max}(\Omega^k)\}; \)

7: \( P_e(\Gamma_m, x_g^{k-1}) \) \( \leftarrow 0 \);

8: \( \text{else} \)

9: \( \text{if } P_e(\Gamma_m, x_g^{k-1}) - \Lambda_c^{max}(\Omega^k) \geq \hat{p}_{ep}(x_g^k) \) then

10: \( \Lambda_c(\Omega_o) \) \( \leftarrow \Lambda_c(\Omega_o) + \Lambda_c^{max}(\Omega^k) \);

11: \( P_e(\Gamma_m, x_g^k) \) \( \leftarrow \hat{p}_{ep}(x_g^k) \);

12: \( \text{else} \)

13: \( \text{if } P_e(\Gamma_m, x_g^{k-1}) - \Lambda_c^{max}(\Omega^k) \leq 0 \) then

14: \( \Lambda_c(\Omega_o) \) \( \leftarrow \Lambda_c(\Omega_o) + \Lambda_c^{max}(\Omega^k) \);

15: \( P_e(\Gamma_m, x_g^{k-1}) \) \( \leftarrow P_e(\Gamma_m, x_g^k) - \Lambda_c^{max}(\Omega^k) \);

16: \( \text{else} \)

17: \( \Lambda_c(\Omega_o) \) \( \leftarrow \Lambda_c(\Omega_o) + P_e(\Gamma_m, x_g^{k-1}) \);

18: \( P_e(\Gamma_m, x_g^{k-1}) \) \( \leftarrow 0 \);

19: \( \text{end if} \)

20: \( \text{end if} \)

21: \( k \leftarrow k + 1 \);

22: \( \text{end while} \)

23: \( \Lambda_c^{opt}(\Omega_o) \) \( \leftarrow \Lambda_c(\Omega_o) \);

As is illustrated in Algorithm 1, we establish each expected battery energy consumption rate \( \Lambda_c(\Omega^k) \) for subregion \( \Omega^k \) (separated by minimum points \( x_g^k \) as is shown in Figure 4) successively, which is determined by three factors: (i) the effective power of \( \Gamma_m \) after its crossing through the previous subregion; (ii) the maximum energy providing ability of the last minimum point; and (iii) whether \( \Omega^k \) contains the zero power point or not. Finally, summing up all \( \Lambda_c(\Omega^k) \) we obtain the total consuming energy \( \Lambda_c^{opt}(\Omega_o) \).

With little modification, Algorithm 1 can be also applied to compute \( \Lambda_h^{opt}(\Omega_i) \). We omit it here to save space.

**IV. MULTIPLE SOURCES CASE**

In this section, we attempt to extend the results to multiple sources case.

For this case, we assume that there are totally \( N \) randomly deployed one-dimensional sources networks, as is illustrated
in Figure 5. The whole area is partitioned into multiple regions which can be classified into two types according to whether nodes within this region can receive a power no less than $p_s$, as depicted in dark color and light color blocks respectively. We denote them as $\Omega^p_h$ ($p = 1, 2, 3, \ldots, P$) and $\Omega^q_c$ ($q = 1, 2, 3, \ldots, Q$) respectively. Similar to Theorem 3.2, we have the lower bound in one dimensional case.

**Theorem 4.1:** Given $N$ sources in region $\Omega$, the tight lower bound of QoEP in one dimensional case is:

$$\begin{align*}
\text{QoEP}_{\text{min}} &= \sum_{p=1}^{P} \int_{\Omega^p_h} f_{dis}(x) dx + \sum_{q=1}^{Q} \int_{\Omega^q_c} f_{dis}(x) \frac{p_r(x)}{p_s} dx
\end{align*}$$

(39)

where $\Omega^p_h$ ($p = 1, 2, 3, \ldots, P$) are regions that can receive a power no less than $p_s$ and not while $\Omega^q_c$ ($q = 1, 2, 3, \ldots, Q$) are regions not, $p_r(x)$ is the cumulative recharge power node receives at $x$.

Its proof is very similar to Theorem 3.2, so we omit it here to save space. In fact, we can further extend the result to 2D and 3D case by applying similar analytical approach in proof of Theorem 3.2.

**Theorem 4.2:** Given $N$ sources in region $\Omega$, the tight lower bounds of QoEP in 2D and 3D cases are:

$$\begin{align*}
\text{QoEP}_{\text{min}} &= \int_{\Omega} f_{dis}(x,y) J(x,y) \frac{p_r(x,y)}{p_s} dxdy
\end{align*}$$

(40)

and:

$$\begin{align*}
\text{QoEP}_{\text{min}} &= \int_{\Omega} f_{dis}(x,y,z) J(x,y,z) \frac{p_r(x,y,z)}{p_s} dxdydz
\end{align*}$$

(41)

respectively, where $J(x) = x$ for $0 \leq x \leq 1$ and 1 for $x > 1$.

**Proof:** We are concerned with 2D case first. Suppose that the whole region interest $\Omega$ can be divided into two subregions, i.e., region $\Omega_c$ wherein node receive power is no less than $p_s$ and $\Omega_h$ wherein node receive power is not. In addition, we define an expansion region $\Omega_e$ wherein node can probably enjoy a nonzero battery energy. That is, for any point $e$ in $\Omega_e$, there exists a path $p$ originated from some point $s$ on boundary of $\Omega_h$ to $e$, followed by a node which starts with a fully charged battery at $s$ and ends with nonzero residual battery energy, while its speed is no more than $v_{\text{max}}$. Then we have:

$$\begin{align*}
\text{QoEP} &= \int_{\Omega_h} f_{dis}(x,y) dxdy + \int_{\Omega_e - \Omega_h} \frac{p_r(x,y)}{p_s} f_{dis}(x,y) dxdy \\
&\quad + \int_{\Omega_e} \text{QoEP}(x,y) f_{dis}(x,y) dxdy
\end{align*}$$

(42)

Since $\text{QoEP}(x,y) \geq \frac{p_r(x,y)}{p_s} f_{dis}(x,y)$ for point $x \in \Omega_h$, it shows that $\text{QoEP}_{\text{min}}$ is indeed a lower bound. Next we continue to prove its tightness.

Similar to proof of Theorem 3.2, we can construct a new mobility model $\mathcal{M}'$, by slowing down the speed at any time with a constant factor $c$ based on an arbitrary mobility model $\mathcal{M}$. Apparently $\mathcal{M}$ and $\mathcal{M}'$ obey the same spatial distribution $f_{dis}(x,y)$. Then its QoEP is subject to:

$$\begin{align*}
\text{QoEP} - \text{QoEP}_{\text{min}} &= \int_{\Omega} (\text{QoEP}(x,y) - \frac{p_r(x,y)}{p_s}) f_{dis}(x,y) dxdy \\
&\leq c_1 \int_{\Omega} 1 dxdy \\
&= c_1 |\Omega_e|
\end{align*}$$

(43)

where $c_1 = \max\{\varepsilon/c_1, |\Omega_e|\}$ based on the properties of continuous function. Consequently, we have:

$$\begin{align*}
\text{QoEP} - \text{QoEP}_{\text{min}} &= c_1 |\Omega_e| \leq \varepsilon
\end{align*}$$

(44)

The result follows.

The analysis is the same for 3D case, so we omit it to save space.

Nevertheless, the calculation of upper bound in multiple case turns to be much more complicated. For example, suppose that the optimal expected battery energy consumption rate for region $\Omega^p_h$ in Figure 5 is $\Lambda_{\text{opt}}^{p}\left(\Omega^p_h\right)$ in its left side and $\Lambda_{\text{opt}}^{c}\left(\Omega^p_h\right)$ in its right side, which are obtained by launching main flows from the left end point and the right end point into $\Omega^p_h$. Then we claim that $\Lambda_c\left(\Omega^p_h\right) \leq \min\{\Lambda_{\text{opt}}^{p}\left(\Omega^p_h\right), \Lambda_{\text{opt}}^{c}\left(\Omega^p_h\right), \Lambda_{\text{opt}}^{c}\left(\Omega^p_h\right)\}$. If one of its end point lies on the boundary of $\Omega$, we set the expected battery energy consumption rate on the corresponding side to 0. For instance, we set $\Lambda_{\text{opt}}^{c}\left(\Omega^p_h\right) = 0$ for region $\Omega^p_h$.

Similarly, it is also hold that $\Lambda_h\left(\Omega^p_h\right)$ for region $\Omega^p_h$ is subject to: $\Lambda_h\left(\Omega^p_h\right) \leq \min\{\Lambda_{\text{opt}}^{h}\left(\Omega^p_h\right), \Lambda_{\text{opt}}^{h}\left(\Omega^p_h\right), \Lambda_{\text{opt}}^{c}\left(\Omega^p_h\right)\}$.

**Theorem 4.3:** Given $N$ sources in region $\Omega$, the QoEP in one dimensional case is subject to:

$$\begin{align*}
\text{QoEP} \leq \text{QoEP}_{\text{min}} + \min\{\sum_{p=1}^{P} \Lambda_{\text{opt}}^{p}\left(\Omega^p_h\right), \sum_{q=1}^{Q} \Lambda_{\text{opt}}^{c}\left(\Omega^p_h\right)\}
\end{align*}$$

(45)

where $\Lambda_{\text{opt}}^{h}\left(\Omega^p_h\right) = \min\{\Lambda_{\text{opt}}^{h}\left(\Omega^p_h\right), \Lambda_{\text{opt}}^{h}\left(\Omega^p_h\right), \Lambda_{\text{opt}}^{c}\left(\Omega^p_h\right)\}$ and $\Lambda_{\text{opt}}^{c}\left(\Omega^p_h\right) = \min\{\Lambda_{\text{opt}}^{c}\left(\Omega^p_h\right), \Lambda_{\text{opt}}^{c}\left(\Omega^p_h\right), \Lambda_{\text{opt}}^{c}\left(\Omega^p_h\right)\}$. 

Figure 5. Illustration of Multiple Randomly Deployed Sources
Note that the right side of inequality (45) is not a tight bound for QoEP, as its flow pattern remains unclear. Furthermore, suppose sources are evenly distributed in one dimensional space with distance \( d \) between neighboring nodes, and the spatial distribution is identical for subregions partitioned by sources and symmetric, as Figure 6 illustrates. By excluding the recharge power from other sources outside, we can decompose the problem and consider only one subregion, which greatly simplifies the problem. Clearly, the problem can be ultimately transformed into that in one source case. The tight upper bound can be obtained in the form similar to Equation (38). We omit the formal statement here to save space.

As we can see in the next section, simulation results demonstrate that our solution outperforms that of path energy provisioning in terms of QoEP estimation and distance estimation for sources deployment.

V. SIMULATION RESULTS

In this section, we present simulation results to verify our findings. Particularly, we consider three mobility models as follows. Table II lists the default simulation parameters for both issues.

A. Single Source Case

We consider two mobility models in this case.

1) Random Waypoint Mobility Model: C. Bettstetter [22] proved that the spatial distribution for random waypoint mobility model (RWMM) in one dimensional case is given by \( f_{dx}(x) = -\frac{3}{x_m^3} x^2 + \frac{3}{4x_m} \) for \(-x_m \leq x \leq x_m\) [23], as is illustrated in Figure 7(a). Comparison of the simulation data points on the dotted curve with the theoretical data points on the solid curve shows they are in good agreement, as we set \( x_m = 1 \) and randomly select speed \( v \) from [0.01, 1]. The pause time for mobile nodes has been fixed as 0, which means nodes are always moving during the entire simulation period. Accordingly we have \( x_T = 0.4257 \).

As shown in Figure 7(b), if \( E_x \) decreases to 0, the simulation results of QoEP approach to the lower bound 0.8519. The ratio in Figure 7(b) refers to \( v_{\text{max}} / v_{\text{min}} \) while \( v_{\text{max}} \) is kept constant and equals to 0.1. Note that the speed of a node for each movement is randomly selected from \([v_{\text{min}}, v_{\text{max}}]\), then different ratio actually leads to different RWMM. Though QoEPs increase monotonically with \( E_x \) for RWMMs of different ratios, they are always bounded by the upper and lower bounds. Specifically, for ratio=1, 2, 10 and 100, the percentage difference between their QoEPs and the upper bound rises from 0.19% to 4.47% near \( 10^{-3} \), and from 0.01% to 7.29% at \( 10^{-2} \). This observation is consistent with our conclusion that the faster the node moves, the larger QoEP it may gain. Moreover, the RWMM of ratio=1, which implies that the node moves with an invariable speed \( v_{\text{max}} \), yields an QoEP very close to the upper bound. This is largely due to the overall random destination selection policy that makes the node incline to move globally, thus speeds up the energy exchange.

Likewise, the QoEPs also increase monotonically with the varying maximum speed \( v_{\text{max}} \) with a constant battery capacity \( E_x = 0.001 \) as Figure 7(c) exhibits. It can be observed that the influence of maximum speed on QoEP is the same as that of battery capacity. This is because accelerating the speed will result in a decrease of stored energy, which is equivalent to increase the battery capacity at a same ratio in terms of the portion of stored energy to energy capacity. As a result, the frequencies of occurrence of energy depletion are the same for both cases, which leads to a same QoEP.

2) Multi-State Mobility Model: We proceed to evaluate our findings in a more complicated case. Suppose that a node moves within region \([-0.9, 0.9]\) following a so-called multi-state mobility model (MSMM). In particular, it turns to the positive direction of the \( x \) axis according to a constant probability \( p(x) = -0.9, -0.7, -0.5, -0.3, -0.1, 0.1, 0.3, 0.5, 0.7, 0.9 \) when it reaches location \( x = 1, 0.2, 0.8, 0.2, 0.8, 0.2, 0.8, 0.2, 0.8, 0.8, 0 \), and to the opposite direction with the remaining probability. Meanwhile, the node changes its speed to a randomly selected value in \([v_{\text{min}}, v_{\text{max}}]\). At the other locations, the node moves with a constant direction and speed.

Apparently, a node’s transition between spatial regions obeys a Markov chain with these 9 subregions being regarded as 9 states. The spatial distribution is sketched in Figure 8(a) as a piecewise function with only two values, 0.2083 and 0.8333. With a fixed \( v_{\text{max}} = 0.1 \) and \( E_x = 0.1 \), the QoEPs with varying battery capacity and speed are demonstrated in Figure 8(b) and Figure 8(c) respectively, which departure from upper bound a larger distance compared with that in RWMM. In Figure 8(b), for example, the percentage difference between QoEPs and the upper bound increases from 12.30% to 19.20% with ratio=1 to 100 near \( 10^{-2} \). This is because the higher tendency of the node to

![Figure 6. Illustration of Multiple Evenly Deployed Sources and Symmetric Spatial Distribution of Node](image-url)
moving locally reduces the opportunity of sufficient energy exchange.

B. Multiple Source Case

For multiple sources case, we are concerned with the random walk mobility model when sources are equidistantly distributed with distance interval $d$. Due to uniform node spatial distribution of this mobility model and symmetry of the sources, we can consider only a subregion $[-d/2, d/2]$ with a reference source placed at the origin. Accordingly, the mobility model we concerned is converted into the random walk with reflection mobility model (RWRMM) proposed in [20]. For this mobility model, each movement occurs in a constant distance traveled $l = 0.4d$, at the end of which a new direction and speed are calculated. If node reaches boundary, it “bounces” off the border and continues along this new direction. It is obvious that RWRMM also follows uniform distribution.

To evaluate the performance of our work, we compare it with that presented in [9]. Specifically, we adapt the approach in [9] to one-dimensional case, by applying the following equation to determine the distance $d$ between adjacent sources: \( \frac{1}{d} \int_{r=0}^{d} \left( \frac{\pi}{(r+\beta)^2} + \frac{\pi}{(d-r+\beta)^2} \right) dr = p_s \). Solving this equation we obtain \( d = \frac{2\pi}{\beta p_s} - \beta \). Indeed, this is exactly the maximum distance to guarantee path energy provisioning, whose QoEP equals to 1 as [9] implies.

In Figure 9(a) and 9(b), we plot the QoEP to varying energy capacity and maximum speed. It shows that the QoEP always equals to 1 for path energy provisioning, which is referred to as PEP in these figures, as we set \( d = \frac{2\pi}{\beta p_s} - \beta = 3.5059 \). In contrast, the QoEPs of real cases, and even that of upper bound are much smaller especially with a relatively low energy capacity or maximum speed.

We proceed to evaluate the gap between our solution and path energy provisioning. In particular, we vary $p_s$, the constant nodal power consumption for working, and compute the maximum distance to guarantee the QoEP.
equals to 1 in lower bound and upper bound cases as well as that for path energy provisioning. It can be seen from Figure 10 that the distance for upper bound increases as $E_\pi$ or $v_{\text{max}}$ increases. Moreover, the impact caused by increasing $E_\pi$ on QoEP is the same as that caused by increasing $v_{\text{max}}$ at the same ratio. We conclude that our proposed upper and lower bounds are more reasonable than path energy provisioning as it takes into consideration the constraints of node speed and energy capacity. In addition, the results of path energy provisioning are proved to be somewhat optimistic and can be regarded as the upper bound when node has “infinite” speed or energy capacity.

After all, the results of lower bound and upper bound can be used to estimate the distance between sources in applications. For example, we can adopt the lower bound in pessimistic cases, upper bound in optimistic cases and mean of lower bound and upper bound in normal cases. More importantly, they can satisfy the requirements of those applications where a predetermined QoEP threshold must be guaranteed. In this sense, they are more practical compared with that of path energy provisioning.

VI. CONCLUSION

In this paper, we have studied the impact of mobility on energy provisioning in WRSNs, especially in one-dimensional cases involving single source and multiple sources respectively. After the proposition of QoEP which aims to qualify the performance of energy provisioning, our work mainly focuses on the upper and lower bounds in both single source and multiple sources. The theoretical results show that the lower bounds in two cases have nothing to do with node speed and battery capacity. By contrary, the upper bounds are greatly affected by these two factors in a complicated way. Thus we present the conception of flow pattern of mobility model by drawing analogy to flow in physics, to facilitate our analysis. It not only enables us to design an optimal mobility model with the largest QoEP which supports the tightness of upper bound in single source case, but also sheds light on effective upper bounds computation.

We conducted simulations to verify our findings. The results show that our bounds hold, and the upper bound can be closely approached by the Random Waypoint Mobility Model with sustained maximum speed in single source case. Moreover, in comparison to that of path energy provisioning, our solution is proved to be more realistic and practical in multiple sources case. Besides, the simulation results also reveal that the influence of node speed on QoEP is the same as that of battery capacity.

REFERENCES


